General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 4 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. *(2 marks)*

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3\mathrm{i}z + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3\mathrm{i}z + 2z^* = 7 + 8\mathrm{i}$$ *(3 marks)*

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\int_9^\infty \frac{1}{\sqrt{x}}\,\mathrm{d}x$; *(3 marks)*

(b) $\int_9^\infty \frac{1}{x\sqrt{x}}\,\mathrm{d}x$. *(4 marks)*

4 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

(a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. *(2 marks)*

(b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of X and Y. *(2 marks)*

(ii) Draw on **Figure 2** a linear graph relating X and Y. *(2 marks)*

(iii) Estimate the values of a and b. *(3 marks)*

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π. *(5 marks)*

(b) Hence find the smallest **positive** value of x which satisfies this equation. *(2 marks)*

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix **AB**. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

Turn over ▶

7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

(a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C. *(2 marks)*

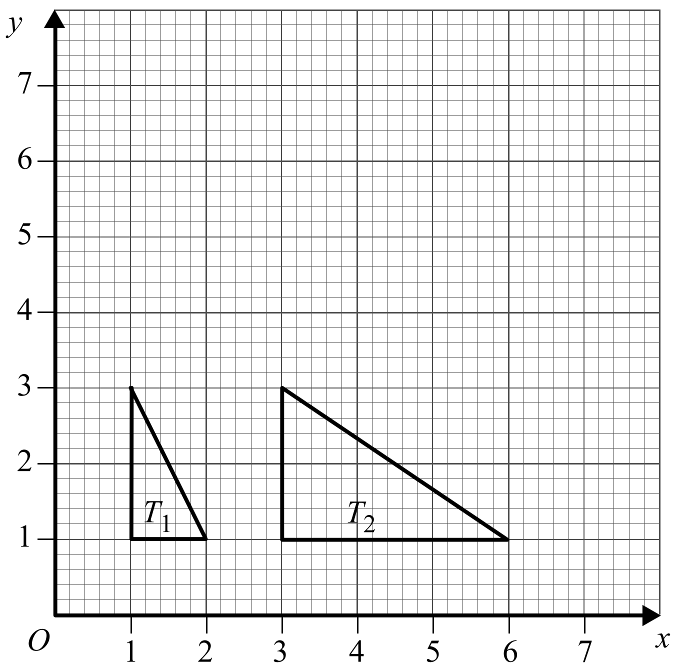
(b) (i) Write down the equations of the two asymptotes of C. *(2 marks)*

(ii) Find the coordinates of the points where the curve C intersects the coordinate axes. *(3 marks)*

(c) Sketch the curve C and its two asymptotes. *(3 marks)*

8 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2.



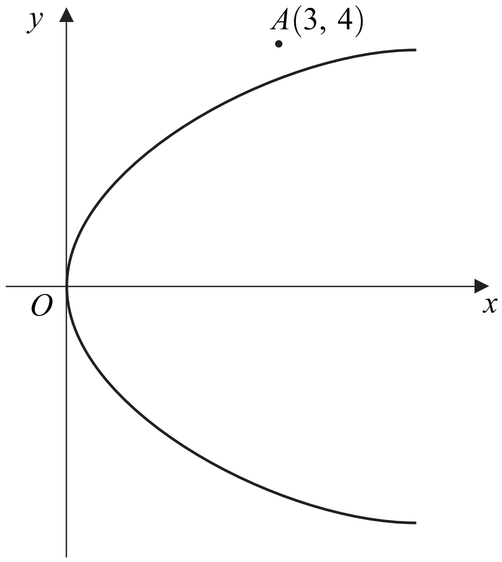
(a) Find the matrix of the stretch which maps T_1 to T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3.

On **Figure 3**, draw the triangle T_3. *(2 marks)*

(c) Find the matrix of the transformation which maps T_1 to T_3. *(3 marks)*

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

END OF QUESTIONS

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Surname		Other Names							
Centre Number						Candidate Number			
Candidate Signature									

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Insert

Insert for use in **Questions 4 and 8**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)

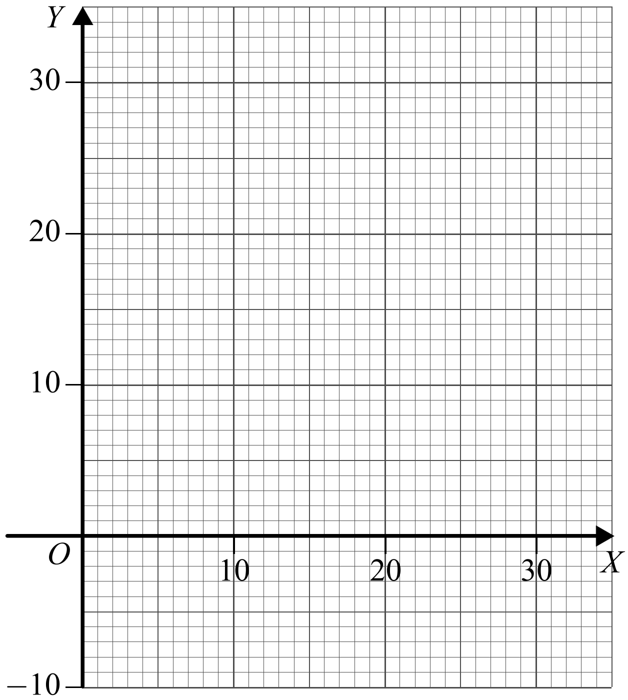
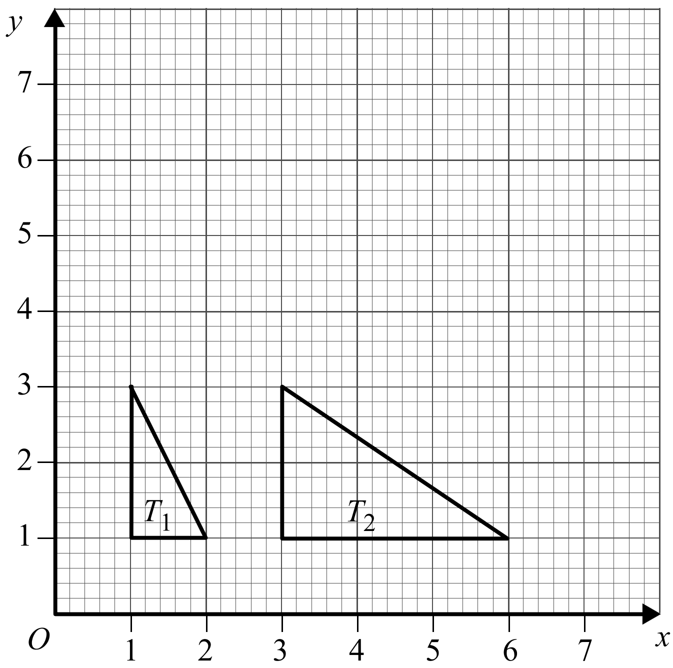


Figure 3 (for use in Question 8)



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